Computational Methods for Inverse Problems in Geophysics

Russell J. Hewett
Mathematics & CMDA, Virginia Tech

Theory and Experience in Solving Inverse Problems in Geophysics Workshop
Uppsala University

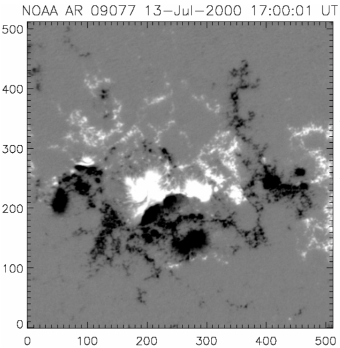
April 9, 2019

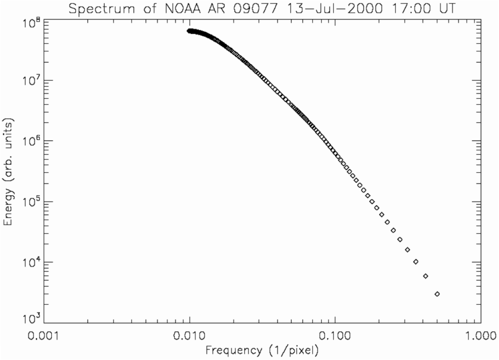
My Background

Research at the intersection of inverse problems, high-performance computation, and physics since ∼2003.

01-05 B.S. in Computer Science from Virginia Tech

- Thesis: Wavelet Analysis of Solar Active Regions
- Topics: Image processing and physical data extraction



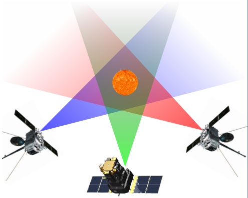


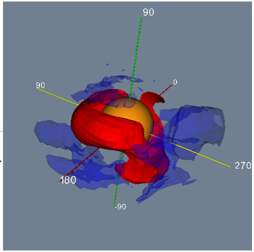
My Background

Research at the intersection of inverse problems, high-performance computation, and physics since ~2003.

05-11 Ph.D. in Computer Science with focus in Computational Science and Engineering from U. of Illinois

- Thesis: Numerical Methods for Solar Tomography in the STEREO Era
- Topics: Dynamic state estimation, constrained Kalman filtering, tomography of solar atmosphere, phase fields, image segmentation, geometrically constrained tomography, ray-tracing, tomography matrix calculation



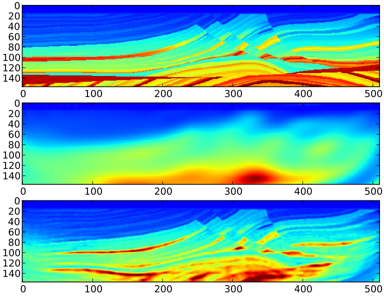


My Background

Research at the intersection of inverse problems, high-performance computation, and physics since ∼2003.

11-14 Postdoc in Mathematics (and Earth Science) at MIT

- Research: Seismic inversion, numerical optimization, wave equation solvers, numerical software design, www.pysit.org



My Background

Research at the intersection of inverse problems, high-performance computation, and physics since ∼2003.

14-18 Research Scientist and Project Manager at Total SA

- Research: High-performance seismic inversion, High-performance software design for CSE activities, geophysical inverse problems, uncertainty quantification, and machine learning

My Background

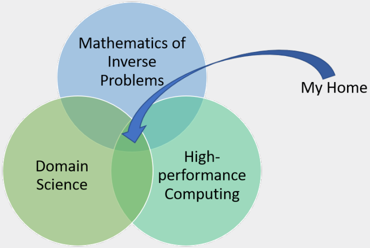
Research at the intersection of inverse problems, high-performance computation, and physics since ∼2003.

18- Assitant Professor in Mathematics and CMDA

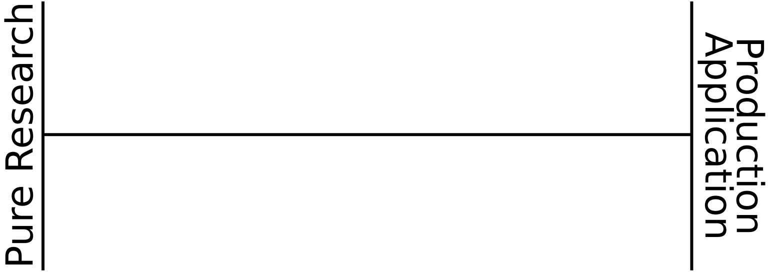
- Research: Machine Learning ∩ Physical Inverse Problems; Inverse Problems at Exascale

Characteristics of Interesting Problems

- Physics based
- Large, *noisy* real data sets
- Extremely large computation requirements



My Research Experience



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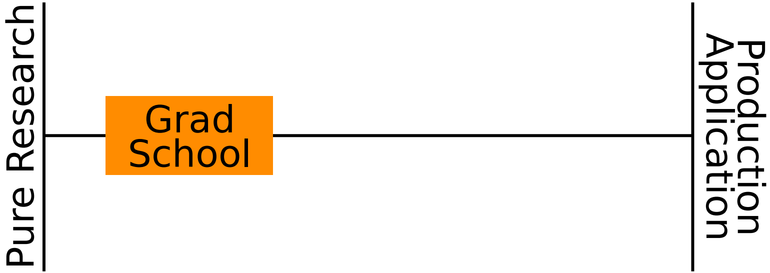

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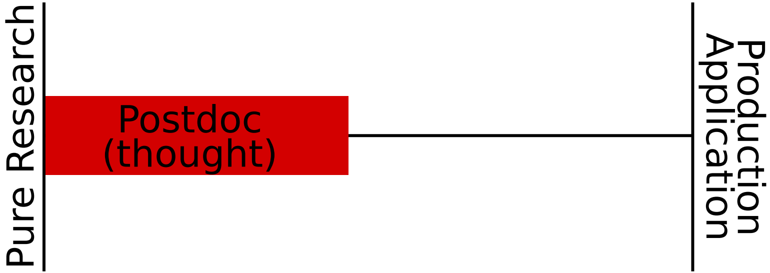

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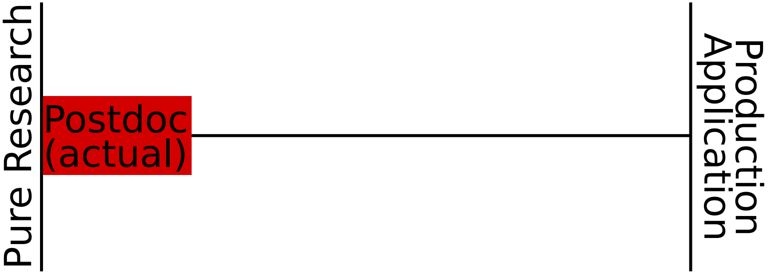

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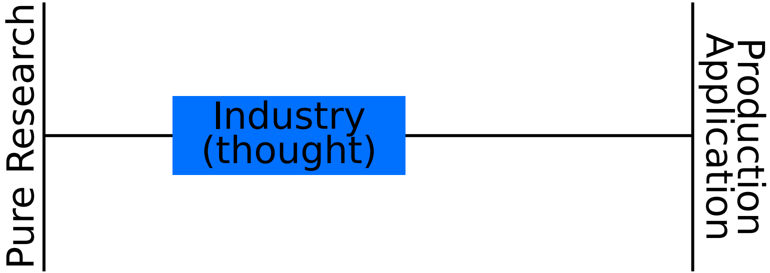

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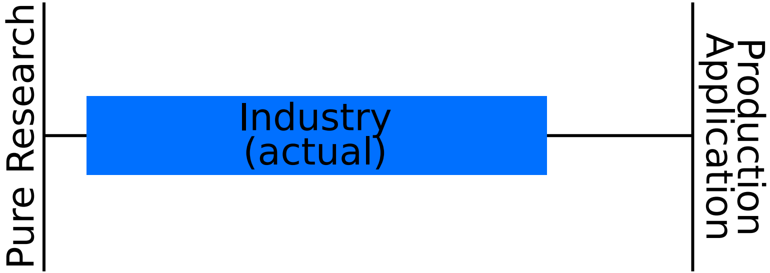
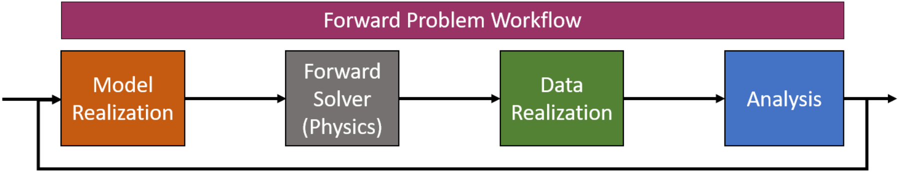


Inverse Problems

Forward Problem

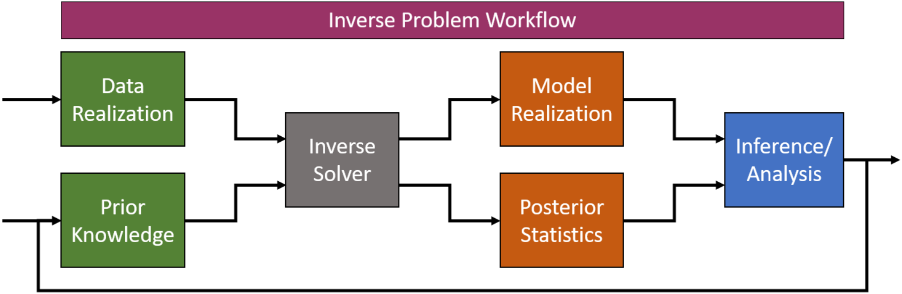
- Need high quality models, usually not empirical
- Need lots of compute
- One model $\Rightarrow$ One solution



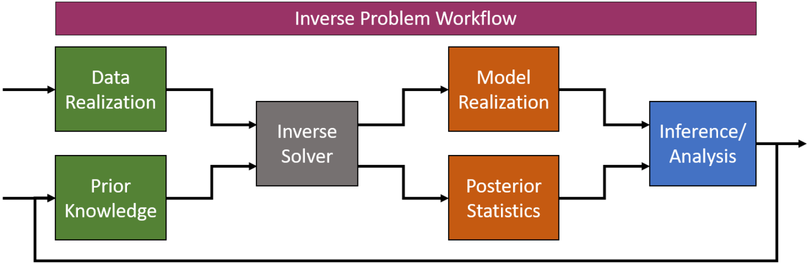
Inverse Problems

Inverse Problem

- Need high-quality data
- Need lots of compute (many fwd problems per inverse problem)
- One data set $\Rightarrow$ space of potential models



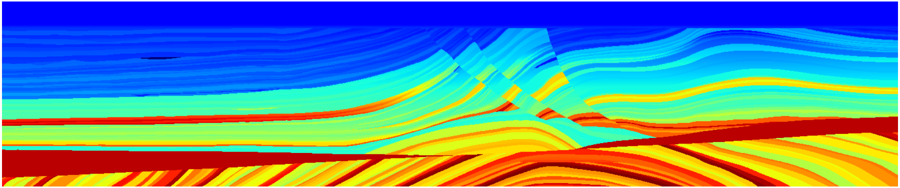
Subsurface Models



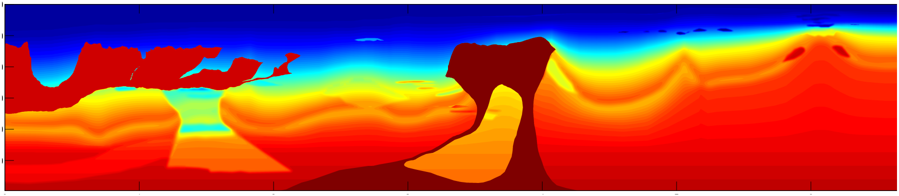
- Subsurface material parameters
  - P-wave velocity (acoustics)
  - S-wave velocity (elastics)
  - Density (both. . . neither?)
  - Anisotropy (VTI, TTI, orthorhombic)
- Subsurface reflectivity
  - Changes in material properties
  - Faults and fractures
- Temporal changes in the above (4D)

Subsurface Models

Marmousi 2 Velocity

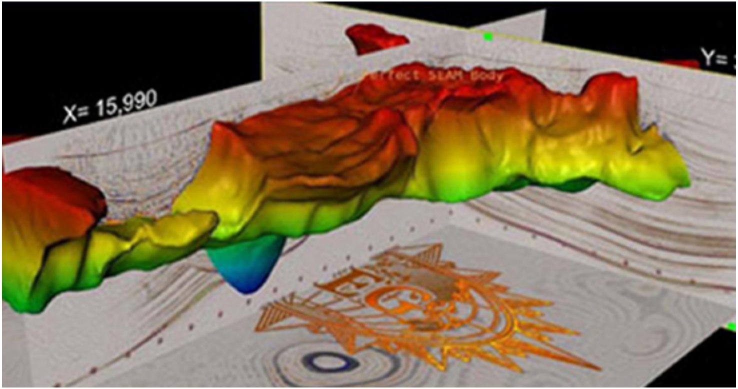


BP 2004 Velocity



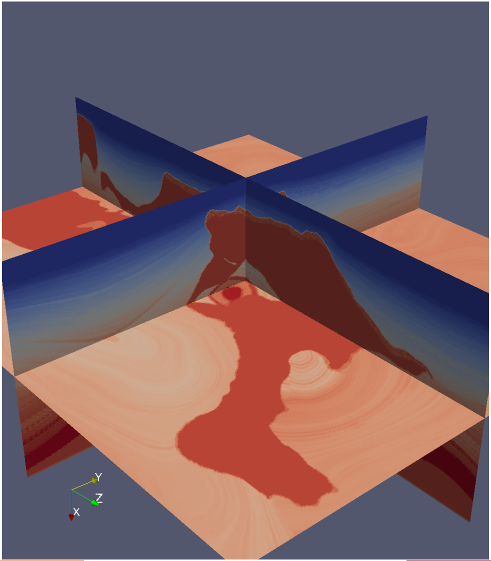
Subsurface Models

SEAM Phase I Velocity (Fehler; SEG)



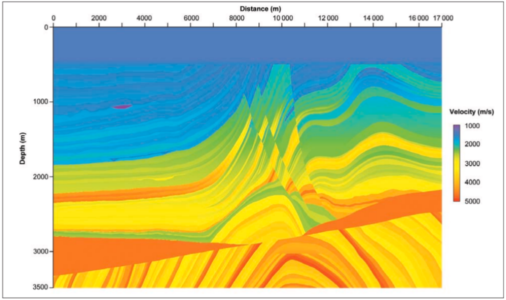
Subsurface Models

SEAM Phase I Velocity



Subsurface Models

Marmousi 2



http://mcee.ou.edu/aaspi/publications/2006/martin_etal_TLE2006.pdf

Subsurface Models

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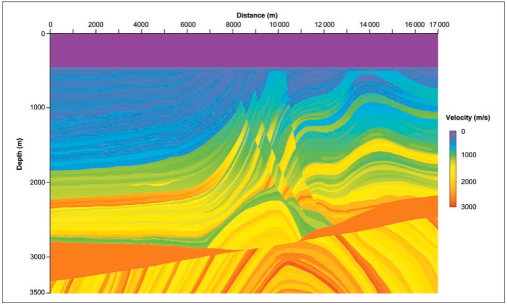

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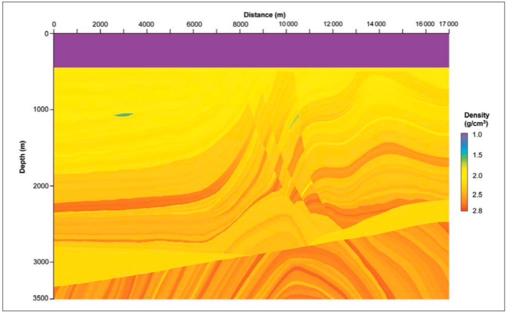
Subsurface Models

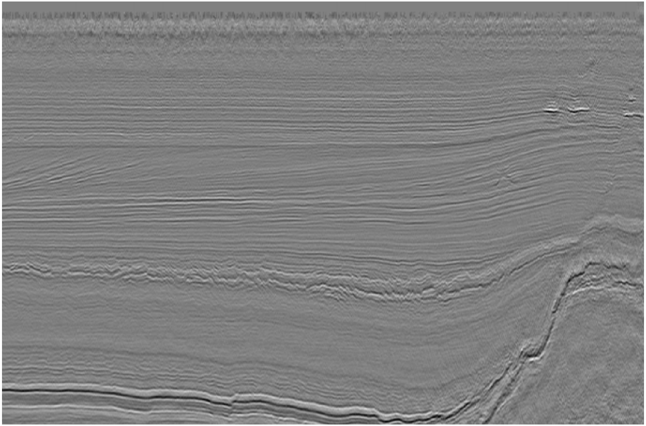
Marmousi 2



http://mcee.ou.edu/aaspi/publications/2006/martin_etal_TLE2006.pdf

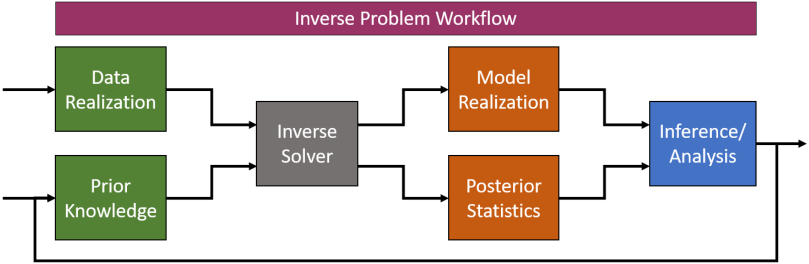
Subsurface Models

Netherlands Block F3 - Crossline 900



https://ghassanalregibdotcom.files.wordpress.com/2018/05/amir_aapg2018_slides.pdf

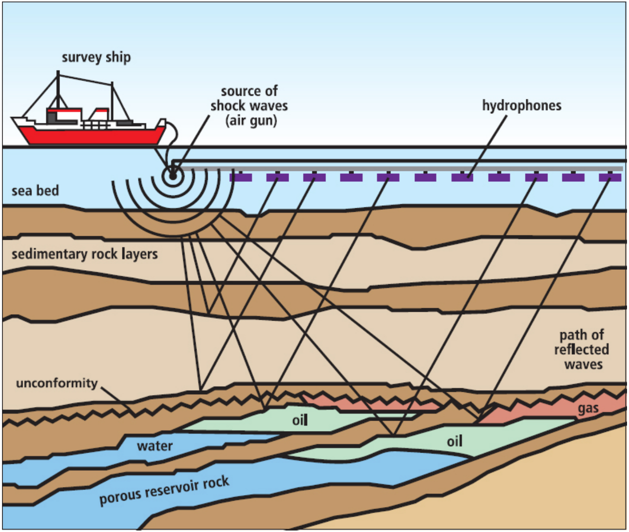
Geophysical Data



- Seismic Survey Design
  - Marine
  - Land
  - Exploration vs monitoring
- Seismic Sources
  - Airguns (marine)
  - Explosive (land)
  - Vibroseis (land, marine)
- Seismic Data Recorders
  - Hydrophones (marine)
  - Geophones (land)
  - DAS

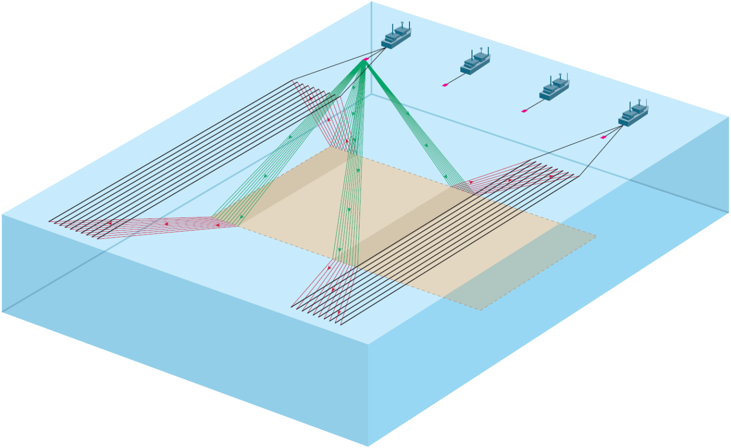
Exploration Seismic Data Acquisition: Design

Marine Survey



Exploration Seismic Data Acquisition: Design

Marine Survey

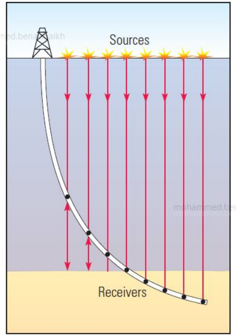


Wide-Azimuth

https://wiki.seg.org/wiki/Wide_azimuth#Wide-azimuth_acquisition_and_survey_design

Exploration Seismic Data Acquisition: Design

Vertical Seismic Profile (VSP) Survey

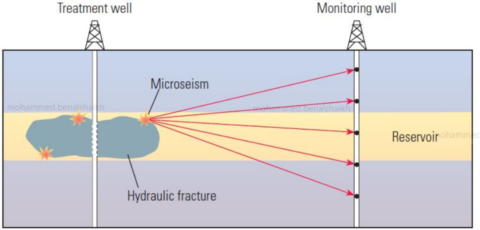


Walk-above

https://wiki.seg.org/wiki/Borehole_geophysics

Exploration Seismic Data Acquisition: Design

Vertical Seismic Profile (VSP) Survey

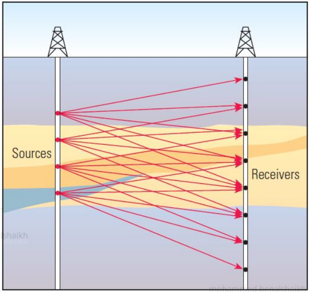


Microseismic

https://wiki.seg.org/wiki/Borehole_geophysics

Exploration Seismic Data Acquisition: Design

Vertical Seismic Profile (VSP) Survey

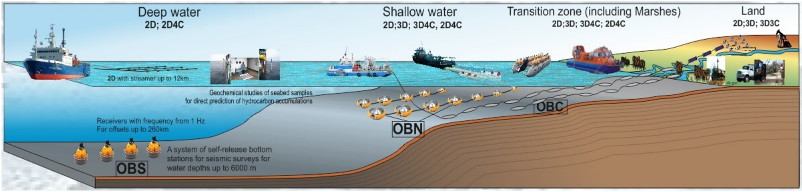


Cross-well

https://wiki.seg.org/wiki/Borehole_geophysics

Exploration Seismic Data Acquisition: Design

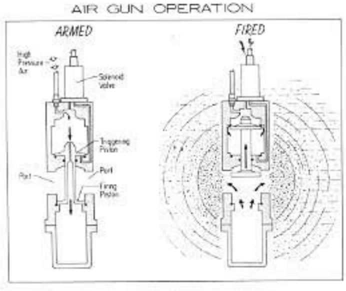
OBC/OBN/OBS Survey



http://marinegeos.com/services/1-seismic-survey

Exploration Seismic Data Acquisition: Sources

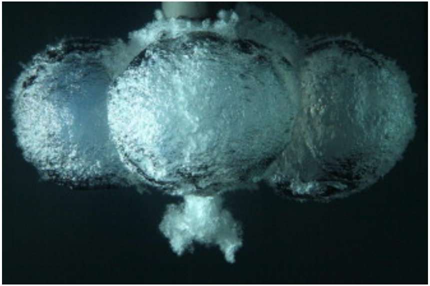
Marine: Airgun



https://woodshole.er.usgs.gov/operations/sfmapping/airgun.htm, Hutchinson and Detrick, 1984

Exploration Seismic Data Acquisition: Sources

Marine: Airgun



https://www.sciencedirect.com/science/article/pii/S0894177714000648

Exploration Seismic Data Acquisition: Sources

Marine: Airgun

https://www.soundingsonline.com/features/atlantic-boaters-may-soon-encounter-seismic-blasting-offshore-drilling

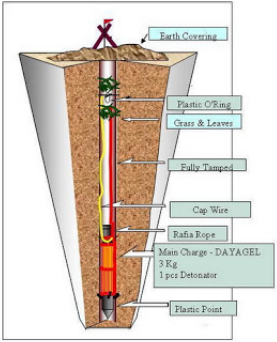
Exploration Seismic Data Acquisition: Sources

Land: VibroSeis

https://pixabay.com/en/vibrator-vibroseis-seismic-survey-863296/

Exploration Seismic Data Acquisition: Sources

Land: Explosive



https://vickybabel10.blogspot.com/2011/03/procedure-seismic-preloading.html

Exploration Seismic Data Acquisition: Sources

Marine: VibroSeis

https://library.seg.org/doi/pdf/10.1190/segam2016-13762702.1

Exploration Seismic Data Acquisition: Receivers

Land: Geophone

http://web.mit.edu/12.000/www/finalpresentation/experiments/geology.html

Exploration Seismic Data Acquisition: Receivers

Land: Geophone

http://web.mit.edu/12.000/www/finalpresentation/experiments/geology.html

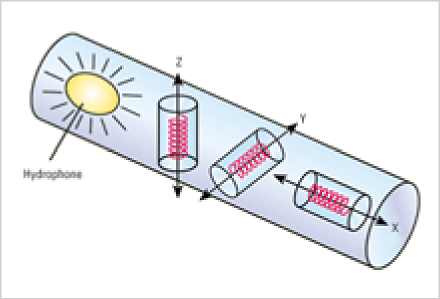
Exploration Seismic Data Acquisition: Receivers

Marine: Hydrophone

https://woodshole.er.usgs.gov/operations/sfmapping/hydrophone.htm

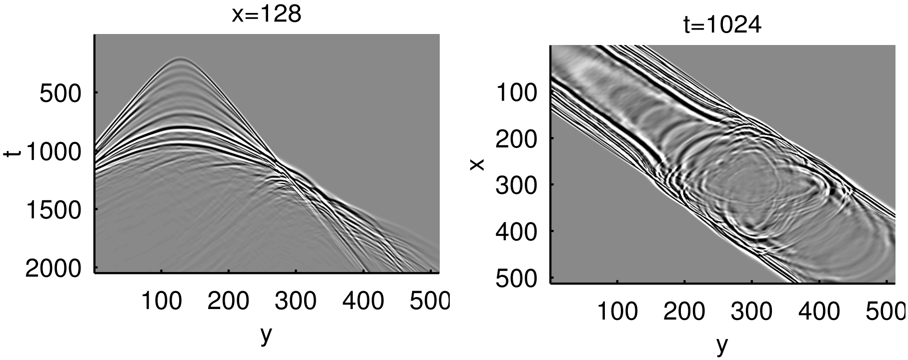
Exploration Seismic Data Acquisition: Receivers

4-Component Sensors (4C)



https://www.glossary.oilfield.slb.com/Terms/sym/4c_seismic_data.aspx

Exploration Seismic Data Acquisition



Geophysical Inverse Problems

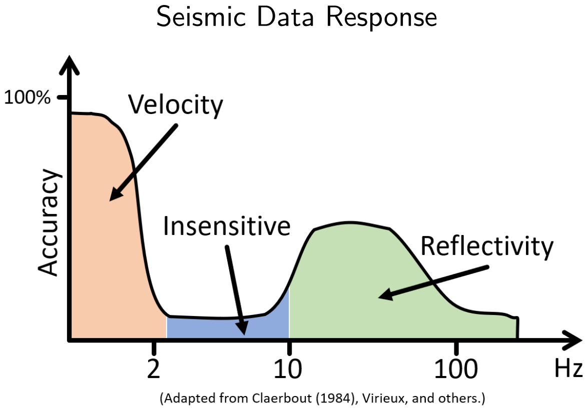
I am considering only the seismic inverse problem.

Regimes I have neglected:

- CSEM (Controlled Source Electromagnetic)
- Gravity
- LIDAR
- etc.

Geophysical Inverse Problems

I will discuss only the full-waveform inversion problem and I will stay in an "exploration" context.



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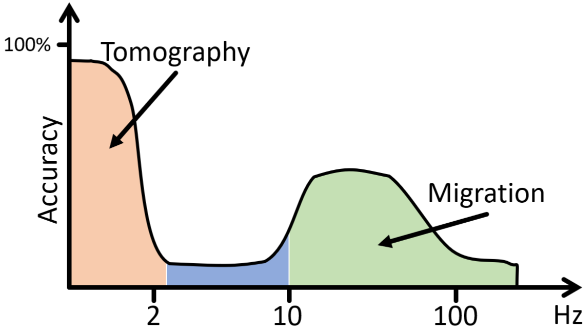
Seismic Data Response



(Adapted from Claerbout (1984), Virieux, and others.)

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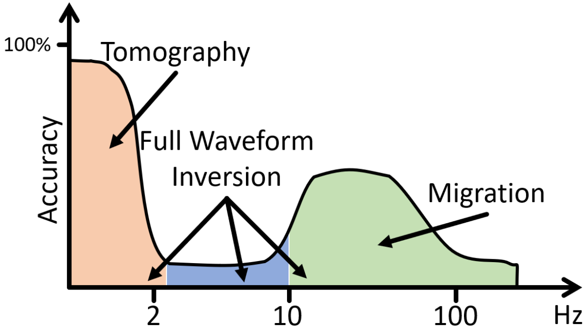

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(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

d: data

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

d: data

k: knowledge

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

d: data

k: knowledge

$\$$: money

(Exploration) Seismic Inverse Problem

$$\max \boldsymbol{\$}(d, k, \$)$$

d: data

k: knowledge

$\$$: money

$\boldsymbol{\$}$: more money

(Exploration) Seismic Inverse Problem

$$\max \boldsymbol{\$}(d, k, \$)$$

Subproblems:

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

Subproblems:

- $\arg\max_{d} \ \mathbf{\$}(d, k, \$)$
  - Find better data
  - Engineers and Analysts

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

Subproblems:

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  - Engineers and Analysts
- $\arg\max_{k} \mathbf{\$}(d, k, \$)$
  - Find better knowledge or use knowledge better
  - Research Scientists

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- $\arg\min_{\$} \max \mathbf{\$}(d, k, \$)$
  - Do all this, but cheaper
  - Managers

(Exploration) Seismic Inverse Problem

$$\max \mathbf{\$}(d, k, \$)$$

Subproblems:

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Seismic Inverse Problem

Full Waveform Inversion Objective

$$J(m) = \|d(t) - \mathcal{F}(m(x))\|_2^2$$

$d(t)$: Data
$m(x)$: Unknown physical coefficients
$\mathcal{F}$: Modeling operator

Seismic Inverse Problem

FWI Objective: "Complete" Version

$$J(m, \boldsymbol{f}) = \sum_{s \in \mathcal{S}} \left\{ \| g(d_s) - g(S_s \mathcal{F}_s(\mathcal{R}_s(m), f_s)) \| + T_f(f_s) \right\} + T_m(m)$$

Seismic Inverse Problem

Full Waveform Inversion Objective

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$d(t)$: Data
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Seismic Inversion

Full Waveform Inversion Objective

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Seismic Inversion

Full Waveform Inversion Problem

$$\arg\min_{m} \; J(m) = \|d(t) - \mathcal{F}(m(x))\|_2^2$$
$$\text{s.t. } L[m]u = f$$
$$\text{for } \mathcal{F}(m) = u$$

- $L[m]u = f$ is a wave equation
- $\mathcal{F}$ operator solves wave equations

- PDE constrained optimization!
  - Time-domain, frequency-domain, Laplace-domain, etc.

Modeling Operators

Example: Second Order Isotropic Acoustics (w/ Constant Density)

- $m(x) = 1/c^2(x)$, where $c(x)$ is p-wave velocity
- L is self adjoint
  - For continuous at least...
  - Up to BCs

$$\mathcal{F}(m_0) = u_0 \quad \Leftrightarrow \quad (m_0\partial_{tt} - \triangle)u_0 = f$$

$$F_{m_0}\delta m = \delta u \quad \Leftrightarrow \quad (m_0\partial_{tt} - \triangle)\delta u = -\delta m \partial_{tt} u_0$$

$$F_{m_0}^* r = \delta m \quad \Leftrightarrow \quad \begin{array}{l} \langle \delta m, -\langle q, \partial_{tt} u_0 \rangle_T \rangle_\Omega \\ \text{s.t.} \quad (m_0\partial_{tt} - \triangle)q = r \end{array}$$

$$\delta m = -\langle q, \partial_{tt} u_0 \rangle_T = -\int_0^T q(x,t)\partial_{tt} u_0(x,t)dt$$

Optimization Setup

$J(m) = \frac{1}{2}||d - \mathcal{F}(m)||_2^2$

- Objective function evaluation
- Computation: Solve wave equation
- Cost: ~ 1

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- Cost: ~ 1

$\nabla J(m_0) = -F_{m_0}^{*}(d - \mathcal{F}(m_0))$

- Objective gradient evalutation
- Computation: Adjoint state method
- Cost: $\sim 2+$

Optimization Setup

$$J(m) = \frac{1}{2}||d - \mathcal{F}(m)||_2^2$$

- Objective function evaluation
- Computation: Solve wave equation
- Cost: ~ 1

$$\nabla J(m_0) = -F_{m_0}^*(d - \mathcal{F}(m_0))$$

- Objective gradient evalutation
- Computation: Adjoint state method
- Cost: $\sim 2+$

$$D^2 J \delta m = F_{m_0}^* F_{m_0} \delta m - < D^2 \mathcal{F} \delta m, d - \mathcal{F}(m_0) >$$

- Objective Hessian application
- Computation: 2^{nd}-order adjoint state method
- Cost: $\sim 4+$

Modeling Operators

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- Forward modeling
- Cost: 1 wave solve

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$$F_{m_0}\delta m = \delta u \quad \Leftrightarrow \quad L[m_0]\delta u = -\frac{\delta L}{\delta m}[\delta m]u_0$$

- Linear forward modeling (Born)
- Cost: 2 wave solves
- Impossible to form as matrix!

Modeling Operators

$\mathcal{F}(m_0) = u_0 \quad \Leftrightarrow \quad L[m_0]u_0 = f \qquad F \in \mathbb{R}^{m \times n}$

- Forward modeling
- Cost: 1 wave solve

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- Forward modeling
- Cost: 1 wave solve

$$F_{m_0}\delta m = \delta u \quad \Leftrightarrow \quad L[m_0]\delta u = -\frac{\delta L}{\delta m}[\delta m]u_0$$

- Linear forward modeling (Born)
- Cost: 2 wave solves
- Impossible to form as matrix!

$F \in \mathbb{R}^{m \times n}$

3D Survey

- 10×1000 rcv (small)
- 8s recording (short)
- 8ms sampling (long)
- m =10,000,000 samples

Modeling Operators

$\mathcal{F}(m_0) = u_0 \quad \Leftrightarrow \quad L[m_0]u_0 = f$

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3D Modeling

- $876 \times 1001 \times 750$ dof
- $n =$ 657,657,000 dof

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Matrix Size

- IEEE single precision. . .
- $\sim$ 23.4PB Storage
- n wave solves
- $\sim$18 years
  @ 100k/day

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$$F^*_{m_0} r = \delta m \quad \Leftrightarrow \quad \begin{array}{c} \left\langle q, -\frac{\delta L}{\delta m}[\delta m]u_0 \right\rangle_{\Omega\times T} \\ \text{s.t.} \quad L^*[m_0]q = r \end{array}$$

- Adjoint modeling
- "Migration" or imaging operator
- Cost: $1+$ wave solves

$F \in \mathbb{R}^{m\times n}$

3D Survey

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Matrix Size

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- $\sim$18 years @ 100k/day

Solving FWI

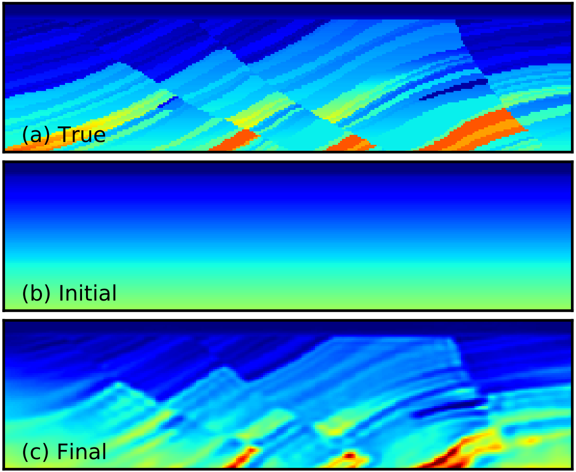
- Solve with gradient-based optimization

Generalized Gradient Scheme

1. Given m_0.
2. While $i < \texttt{MaxIter}$
   - 2.1 $g_i = \nabla J(m_i)$
   - 2.2 $s_i = h(m_i, g_i)$
   - 2.3 $\alpha = \texttt{LineSearch}(m_i, g_i, s_i)$
   - 2.4 $m_{i+1} = m_i + \alpha s_i$

- Gradient descent?
- L-BFGS?
- Hessian/Quasi-Newton schemes?

Solving FWI



- 50 L-BFGS iterations w/ Locally 1D Time Solver (L. Zepeda, RJH, M. Rao, L. Demanet (SEG 2013))

Real-world Utility

- Interpreters don't actually use inverted material parameters
- They still want to look at seismic "images"
- Found by "migration" or imaging algorithms
    - Kirchoff migration
    - (One-way) wave equation migration
    - Reverse-time migration
    - Linearized inversion

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- Or, we can consider it as a gradient calculation in FWI

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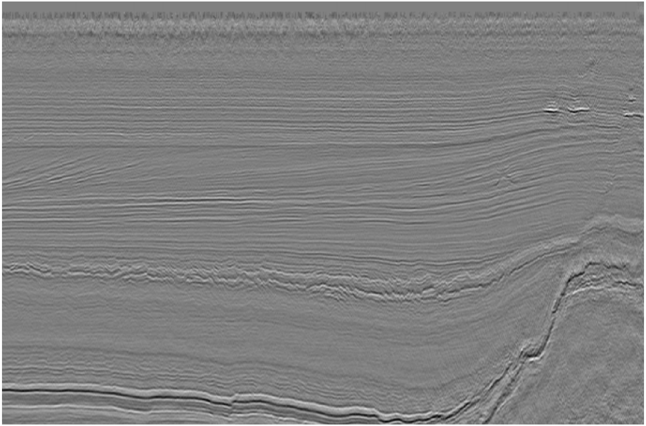
  - Reverse-time migration
  - Linearized inversion
- Migration is essentially back-propagation
- Or, we can consider it as a gradient calculation in FWI
- In any case, it is computed at much higher-frequency
- Looking at reflectivity, not material parameters

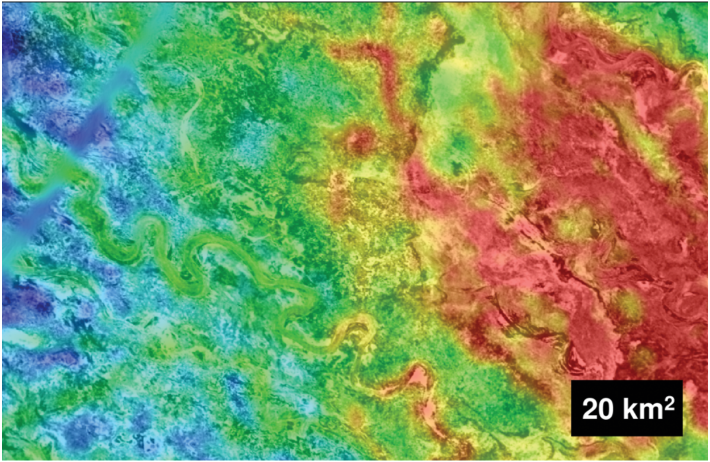
Seismic Image

Netherlands Block F3 - Crossline 900



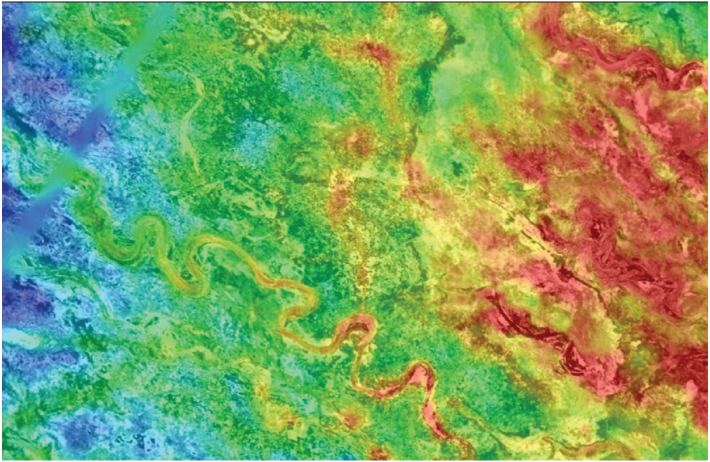
https://ghassanalregibdotcom.files.wordpress.com/2018/05/amir_aapg2018_slides.pdf

Real World Results



"An offshore Gabon full-waveform inversion case study," Xiao, et al., *Interpretation*, November 2016. Data from CGG.

Real World Results



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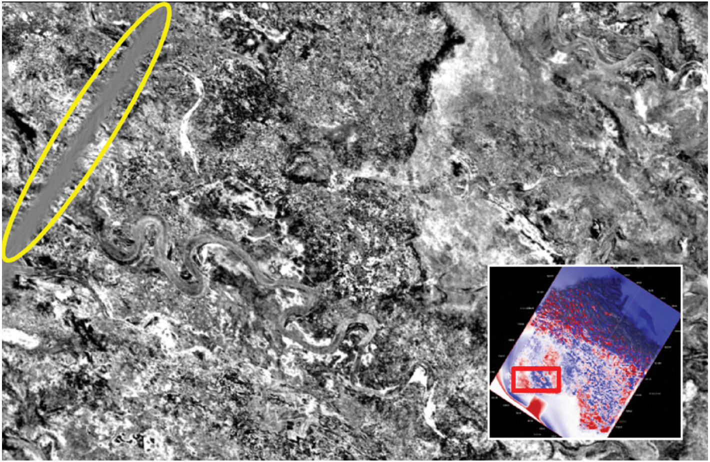

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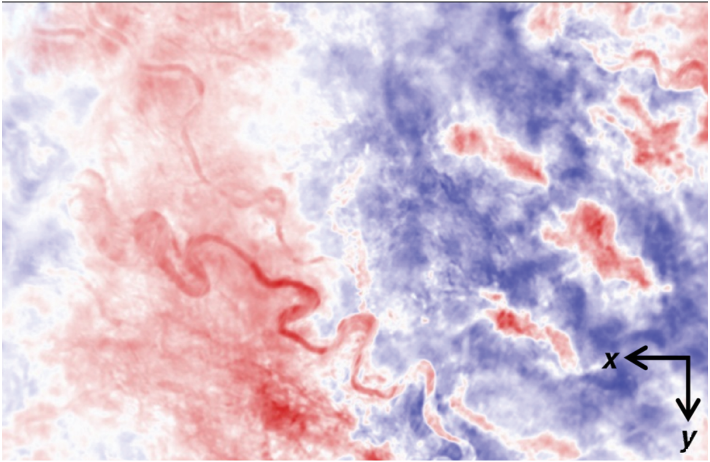
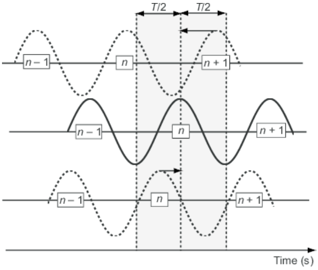


"An offshore Gabon full-waveform inversion case study," Xiao, et al., *Interpretation*, November 2016. Data from CGG.

Challenges for FWI

- Due to the physical formulation
- Due to mathematical formulation
- Due to computational requirements
- Due to business decisions

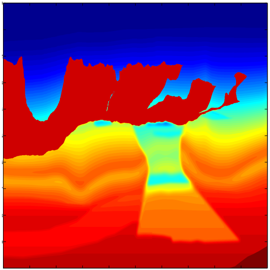
Cycle Skipping



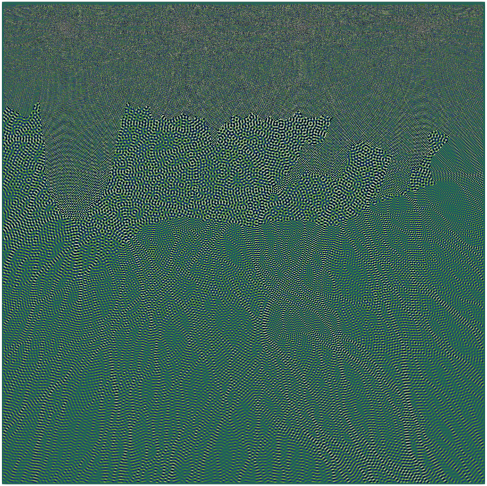
Jean Virieux

- Due to the physical and mathematical formulations
- Manifests as global nonconvexity
- Partially resolved by working in frequency domain (or in Laplace domain)
- Time-domain wave equation becomes Helmholtz equation

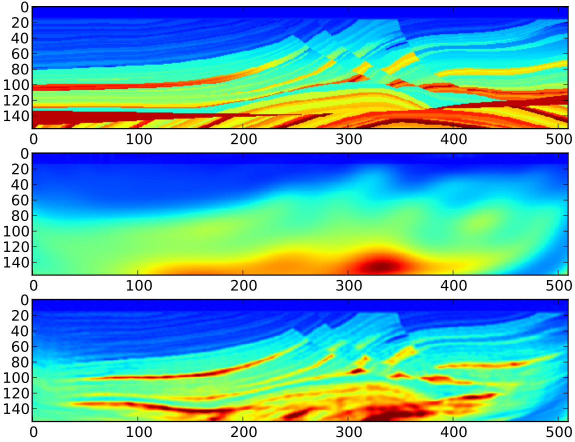
FWI In Frequency Domain



- max. 512 wavelengths in domain
- PML width: 2.5 wavelengths
- 64×64 domain decomposition



FWI In Frequency Domain



- Frequency continuation

Realistic and Practical Physics

- I have shown the *idea* behind FWI using constant density acoustic physics
- Of course, the earth is not constant density, nor acoustic
- Massive increase in computational – and software – costs
- Does better physics drive the need for more compute?
- Or is more compute driving the availability of better physics?

Realistic and Practical Physics

Physical Models

$$\partial_{tt}u = a\triangle u + f$$

Physics	Solutions	Parameters	Computation
Iso-Aco (const. ρ) (2nd)	1	1	–

Realistic and Practical Physics

Physical Models

$$\partial_t \begin{bmatrix} p \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} & -c\nabla\cdot \\ a\nabla & \end{bmatrix} \begin{bmatrix} p \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} g \\ \boldsymbol{h} \end{bmatrix}$$

Physics	Solutions	Parameters	Computation
Iso-Aco (const. ρ) (2nd)	1	1	–
Iso-Aco (1st)	4	2	1x-2x

Realistic and Practical Physics

Physical Models

$$\partial_t \begin{bmatrix} \tilde{\boldsymbol{\sigma}} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} & -\boldsymbol{A}^T \boldsymbol{D}^* \boldsymbol{R} \\ a\boldsymbol{R}^T \boldsymbol{D} \boldsymbol{A} & \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{\sigma}} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{bmatrix}$$

Physics	Solutions	Parameters	Computation
Iso-Aco (const. ρ) (2nd)	1	1	–
Iso-Aco (1st)	4	2	1x-2x
TTI-Aco (1st)	5 (+?)	4	3x

Realistic and Practical Physics

Physical Models

$$\partial_t \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} & -\mathbb{C}\boldsymbol{D}^* \\ a\boldsymbol{D} & \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{bmatrix}$$

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Iso-Ela (1st)	9	3	$\sim$ 40x

Realistic and Practical Physics

Physical Models

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Iso-Aco (1st)	4	2	1x-2x
TTI-Aco (1st)	5 (+?)	4	3x
Iso-Ela (1st)	9	3	$\sim$ 40x
VTI-Ela (1st)	9	8	$\sim$ 40x

Realistic and Practical Physics

Physical Models

$$\partial_t \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} & -\mathbb{C}\boldsymbol{D}^* \\ a\boldsymbol{D} & \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{bmatrix}$$

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Iso-Aco (const. ρ) (2nd)	1	1	–
Iso-Aco (1st)	4	2	1x-2x
TTI-Aco (1st)	5 $(+?)$	4	3x
Iso-Ela (1st)	9	3	$\sim$ 40x
VTI-Ela (1st)	9	8	$\sim$ 40x
TTI-Ela (1st)	9	36	$>$ 100x

Realistic and Practical Physics

Physical Models

$$\partial_t \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} & -\mathbb{C}\boldsymbol{D}^* \\ a\boldsymbol{D} & \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{bmatrix}$$

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TTI-Aco (1st)	5 (+?)	4	3x
Iso-Ela (1st)	9	3	$\sim$ 40x
VTI-Ela (1st)	9	8	$\sim$ 40x
TTI-Ela (1st)	9	36	$>$ 100x
Elastic (1st)	9	81	$>$ 100x

Software Considerations

- I have greatly simplified the problem
- Only considering one simple PDE and an "academic" objective function
- My personal results in this talk are only embarassingly parallel
- Essentially data parallel in shot record
- There is still model parallelism to exploit, which requires high-end HPC software
- In addition to. . .

Software Considerations

FWI Objective

$$J(m) = \|d - \mathcal{F}(m)\|_2^2$$

$d(t)$:	Data
$m(x)$:	Unknown physical coefficients
$\mathcal{F}$:	Modeling operator

Software Considerations

FWI Objective

$$J(m) = \sum_{s \in \mathcal{S}} \|d_s - \mathcal{F}_s(m)\|_2^2$$

$d_s(t)$: Data for shot s
$m(x)$: Unknown physical coefficients
$\mathcal{F}_s$: Modeling operator for shot s

Software Considerations

FWI Objective

$$J(m) = \sum_{s \in \mathcal{S}} \| d_s - S_s \mathcal{F}_s(m) \|_2^2$$

$d_s(t)$: Data for shot s
$m(x)$: Unknown physical coefficients
$\mathcal{F}_s$: Modeling operator for shot s

S_s: Data sampling operator for shot s

Software Considerations

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S_s: Data sampling operator for shot s
$\|\cdot\|$: (Arbitrary) residual norm

Software Considerations

FWI Objective

$$J(m, \boldsymbol{f}) = \sum_{s \in \mathcal{S}} \| d_s - S_s \mathcal{F}_s(m, f_s) \|$$

- $d_s(t)$: Data for shot s
- $m(x)$: Unknown physical coefficients
- $\mathcal{F}_s$: Modeling operator for shot s

- S_s: Data sampling operator for shot s
- $\|\cdot\|$: (Arbitrary) residual norm
- $f_s(x, t)$: Unknown seismic source function for shot s

Software Considerations

FWI Objective

$$J(m, \boldsymbol{f}) = \sum_{s \in \mathcal{S}} \| g(d_s) - g(S_s \mathcal{F}_s(m, f_s)) \|$$

$d_s(t)$:	Data for shot s
$m(x)$:	Unknown physical coefficients
$\mathcal{F}_s$:	Modeling operator for shot s
S_s:	Data sampling operator for shot s
$\|\cdot\|$:	(Arbitrary) residual norm
$f_s(x, t)$:	Unknown seismic source function for shot s
g:	Data filter and interpolation function

Software Considerations

FWI Objective

$$J(m, \boldsymbol{f}) = \sum_{s \in \mathcal{S}} \| g(d_s) - g(S_s \mathcal{F}_s(\mathcal{R}_s(m), f_s)) \|$$

$d_s(t)$:	Data for shot s
$m(x)$:	Unknown physical coefficients
$\mathcal{F}_s$:	Modeling operator for shot s
S_s:	Data sampling operator for shot s
$\|\cdot\|$:	(Arbitrary) residual norm
$f_s(x,t)$:	Unknown seismic source function for shot s
g:	Data filter and interpolation function
$\mathcal{R}_s$:	Model restriction and filter operator for shot s

Software Considerations

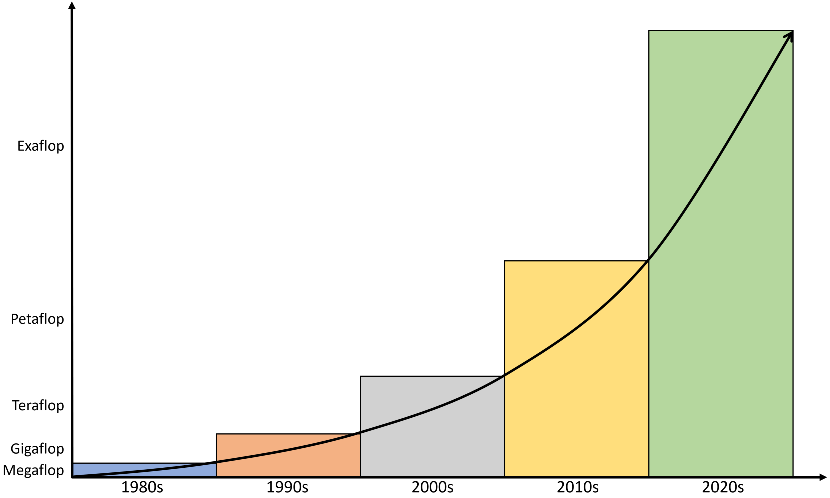
FWI Objective

$$J(m, \boldsymbol{f}) = \sum_{s \in \mathcal{S}} \{\|g(d_s) - g(S_s \mathcal{F}_s(\mathcal{R}_s(m), f_s))\| + T_f(f_s)\} + T_m(m)$$

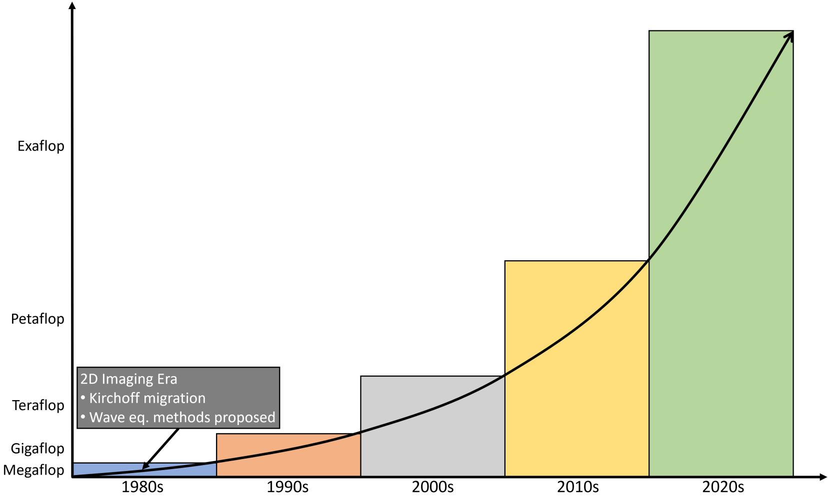
$d_s(t)$: Data for shot s
$m(x)$: Unknown physical coefficients
$\mathcal{F}_s$: Modeling operator for shot s

S_s: Data sampling operator for shot s
$\|\cdot\|$: (Arbitrary) residual norm
$f_s(x,t)$: Unknown seismic source function for shot s
g: Data filter and interpolation function
$\mathcal{R}_s$: Model restriction and filter operator for shot s
T: Regularization

HPC and Subsurface Inverse Problems



HPC and Subsurface Inverse Problems



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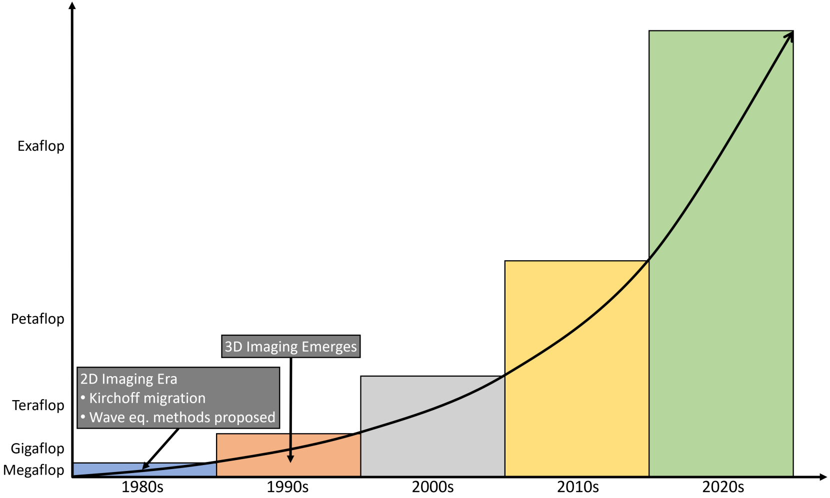

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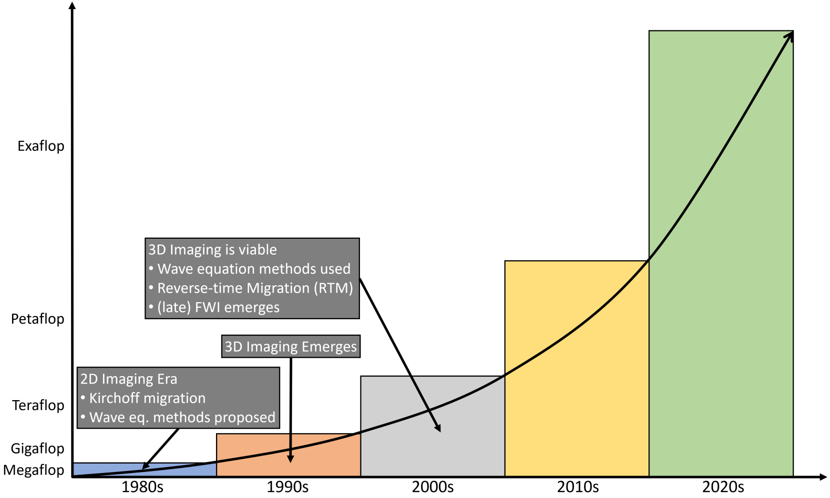
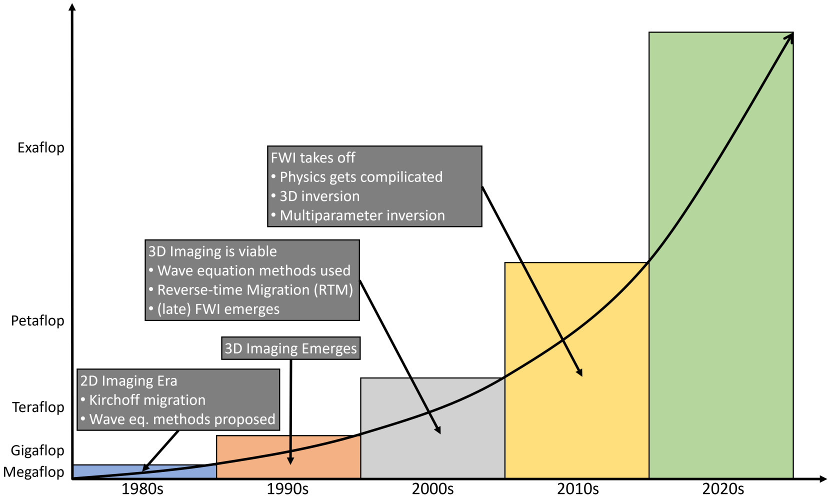
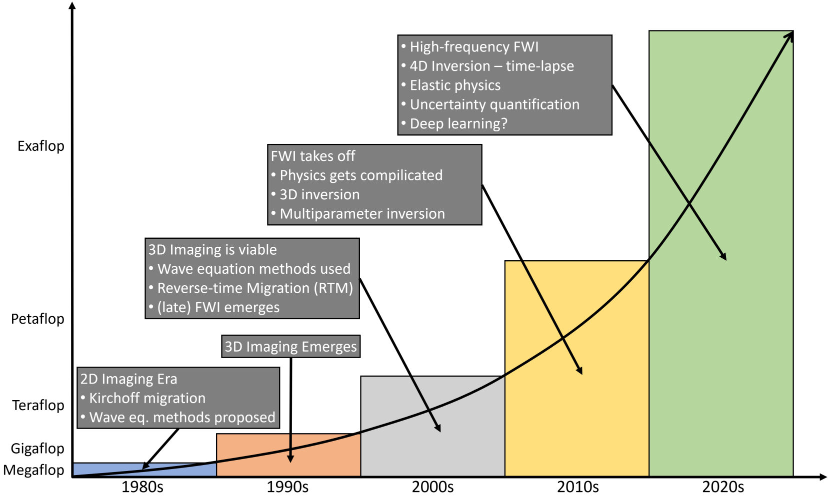


HPC and Subsurface Inverse Problems

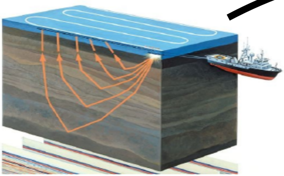


HPC and Subsurface Inverse Problems



Scale: Business Considerations

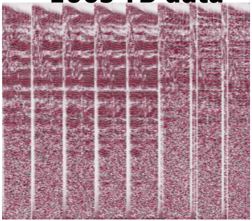
100s M$



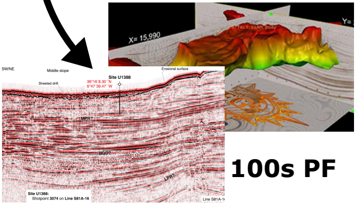
1000s km^2 surface acquisition

Acquire Seismic Data

100s TB data



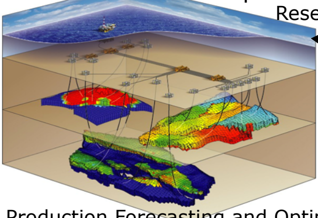
Seismic Depth Imaging



100s PF

???s Gboe ???? M$ Investment

Reservoir Appraisal



Production Forecasting and Optimization

Current / Future Challenges: Data and Acquisition

Field data

- Exploit high redundancy in field data (compression)
- "Chain of custody" in field data
- Real data is noisy!
- Multicomponent data (more than just pressure)

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Novel Acquisitions

- Marine: Coil surveys, multiple sources
- Marine vibrators
- Pseudo-random source/receiver geometry
- Survey refinement

Current / Future Challenges: PDEs and Inverse Problems

Inverse Problems

- Source function is unknown
- Meshing complex topography (seafloor, salts)
- Uncertainty quantification

PDEs

- Boundary conditions!
- Beyond finite difference

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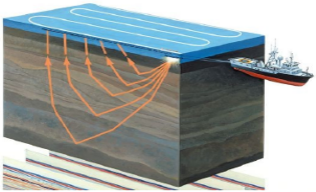

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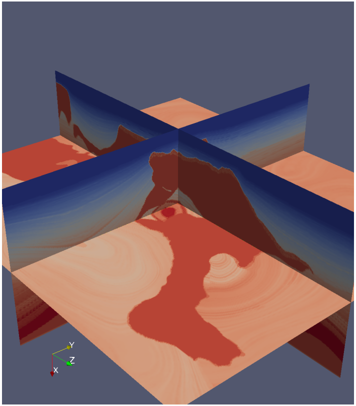

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PDEs

- Boundary conditions!
- Beyond finite difference

- Perfectly Matched Layers
- Anisotropy
- High-order ABCs

Current / Future Challenges: PDEs and Inverse Problems

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- Uncertainty quantification

PDEs

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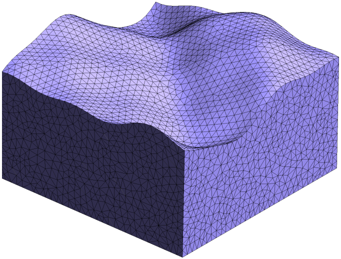
- Beyond finite difference

Example: Discontinuous Galerkin Solvers

- Advantages
  - Can resolve complex topography
  - Natural mapping to accelerators
  - Many tuning parameters
- Challenges
  - Achieving peak performance (beating finite difference)
  - Application in inverse problems
  - Absorbing boundaries
  - Many tuning parameters
- Strategies
  - Hybrid meshing (& adjoints)
  - h- and p-refinement (& adjoints)
  - Local time stepping (& adjoints)
  - Optimal basis functions (& adjoints)



J. Chan, Z. Wang, RJH, T. Warburton (2016)

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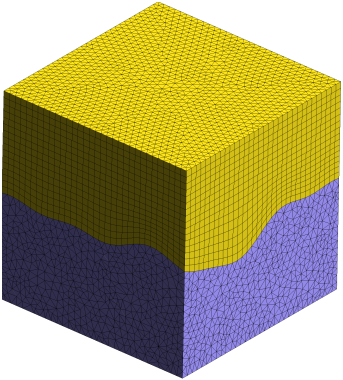

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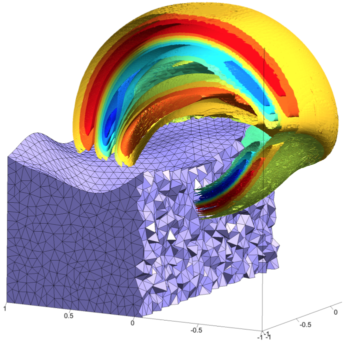

J. Chan, Z. Wang, RJH, T. Warburton (2016)

Example: Discontinuous Galerkin Solvers

- Advantages
  - Can resolve complex topography
  - Natural mapping to accelerators
  - Many tuning parameters
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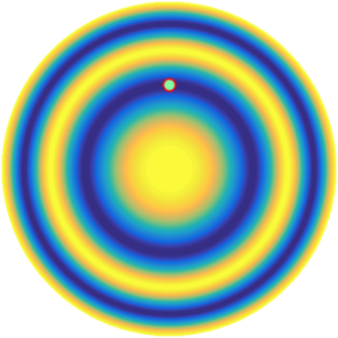

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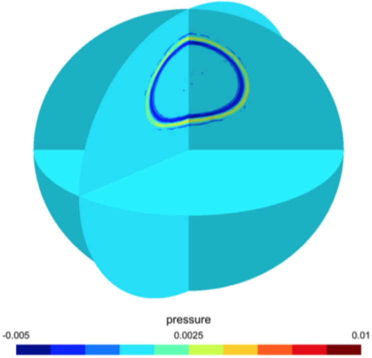

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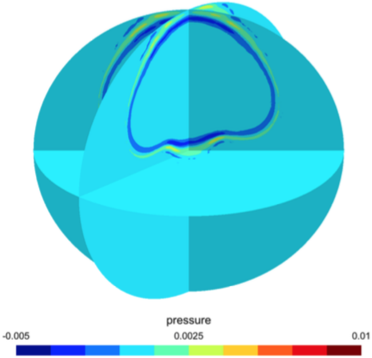

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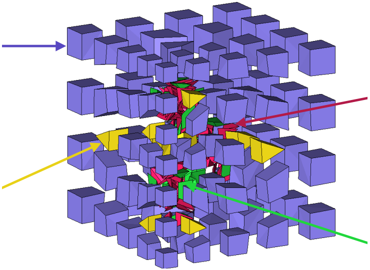

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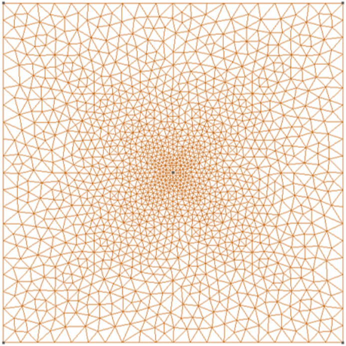

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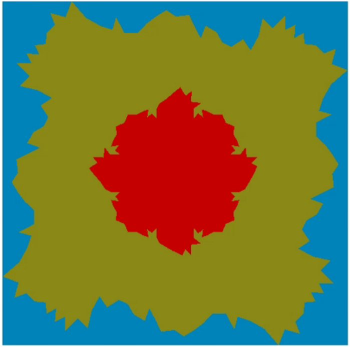

Current / Future Challenges

Optimization for large-scale PDE constrained IPs

- Uncertainty quantification without access to the Hessian
- What parameters can be inverted for?
  - Seismic data is not sensitive to density
  - Shear velocity only visible through S-P conversion
  - What about Thomson parameters?
- Joint inversion with CSEM, gravity, etc.

Current / Future Challenges

HPC & Software engineering

- HPC portable codes
- Math operations obscured deep in legacy codes – software architecture issues
- Domain experts are not expert computational scientists
- Efficient automatic adjoints and derivatives – numerical correctness vs HPC correctness
- Integration with rapidly developing exascale software
- Incredible cost of future algorithms

- CPU only is not viable
- “All” new machines are GPU machines

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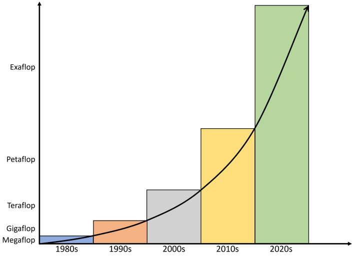

Current and Future Challenges

Machine Learning / Deep Learning

- Flavor of the ~~month~~ ~~year~~ decade?
- "Universal function approximation"
- "High-dimensional interpolation problem" (Mallat)
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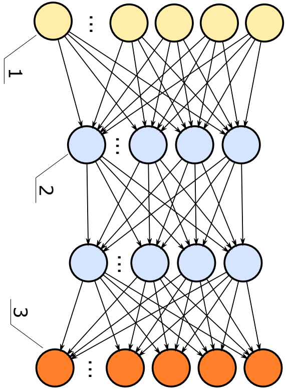

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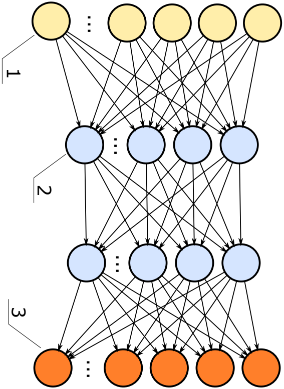


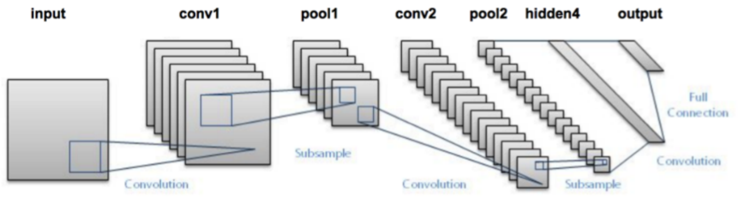
Adjoint States: Least-Squares

$$
\begin{aligned}
J(\boldsymbol{x}) &= \tfrac{1}{2}||\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}||_2^2 \\
\nabla J(\boldsymbol{x}_0) &= -\boldsymbol{A}^*(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x_0}) \\
&= -\boldsymbol{A}^T(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x_0})
\end{aligned}
$$

Adjoint States: Deep Learning

$$
\begin{aligned}
J(\boldsymbol{w},\boldsymbol{b}) &= \tfrac{1}{2}||d - \mathcal{F}(\boldsymbol{w},\boldsymbol{b}; I)||_2^2 \\
&= \tfrac{1}{2}||d - \mathcal{O}_6(\boldsymbol{w}_6,\boldsymbol{b}_6; \mathcal{C}_5(\ldots; \mathcal{P}_4(\ldots; \mathcal{C}_3(\ldots; \mathcal{P}_2(\boldsymbol{w}_2,\boldsymbol{b}_2; \mathcal{C}_1(\boldsymbol{w}_1,\boldsymbol{b}_1; I))))))||_2^2
\end{aligned}
$$

LeNet-5



Adjoint States: Deep Learning

$$\begin{aligned}
J(\boldsymbol{w}, \boldsymbol{b}) &= \tfrac{1}{2}||d - \mathcal{F}(\boldsymbol{w}, \boldsymbol{b}; I)||_2^2 \\
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\nabla J(\boldsymbol{w}_0, \boldsymbol{b}_0) &= -F^*(d - \mathcal{F}(\boldsymbol{w}, \boldsymbol{b}; I)) \\
&= -\boldsymbol{C}_1^* \boldsymbol{P}_2^* \boldsymbol{C}_3^* \boldsymbol{P}_4^* \boldsymbol{C}_5^* \boldsymbol{O}_6^* (d - \mathcal{F}(\boldsymbol{w}, \boldsymbol{b}; I))
\end{aligned}$$

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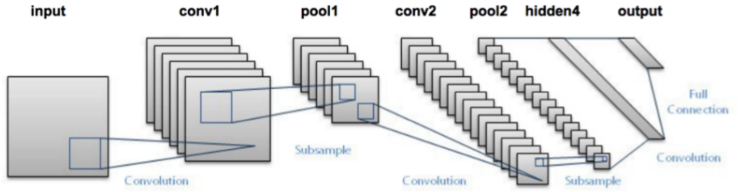

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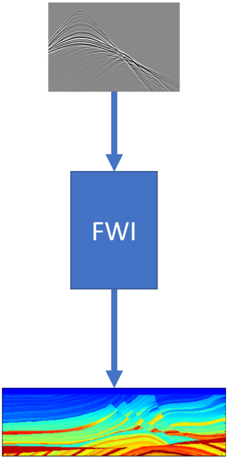

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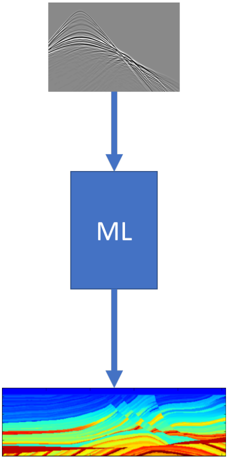

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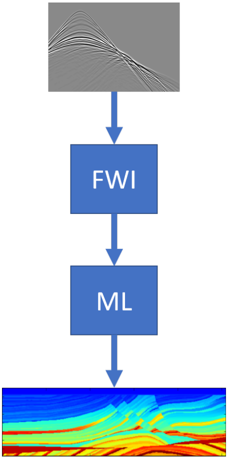

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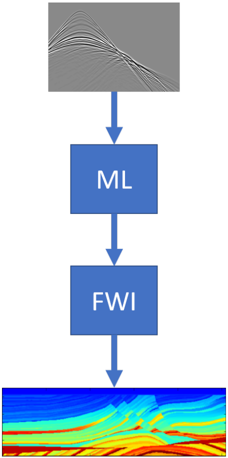

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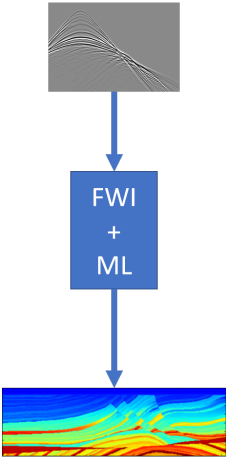

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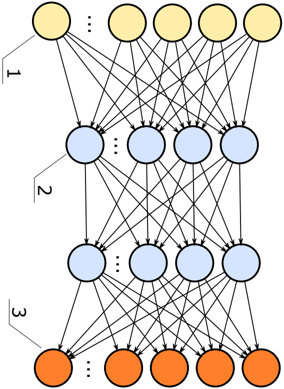

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Summary

Computational Science R&D Flow

physics → math → algorithm → HPC computation

In all of these aspects. . .

- Scale of exploration seismic inverse problems is astounding
- As academics we need to think bigger. . .
  - . . . and train students to think bigger!
- There are lots of interesting and challenging computational, mathematical, and numerical problems arising from seismic inverse problems

Summary

- Give FWI a shot:

  www.pysit.org

- Try the 1D FWI development exercise!