L-sweeps: A scalable parallel high-frequency Helmholtz solver

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SIAM CSE 2019, Spokane, WA

February 28, 2019

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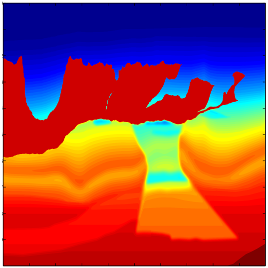
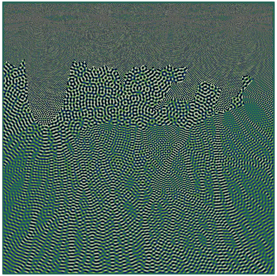
Wave propagation in geophysical applications



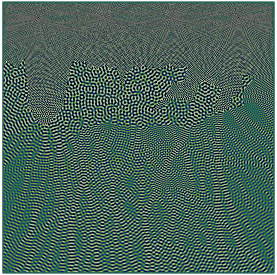
Inhomogeneous media



High frequency

$$-\Delta u - \omega^2 m u = f \quad \text{in } \Omega$$

$$+ \text{ A.B.C. at } \partial\Omega$$



Ω	...	Domain of interest	m	...	squared slowness
ω	...	frequency	f	...	sources

- Classical iterative methods: n_{iter} grows with ω

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- Classical direct methods:

	1D	2D	3D
operations	$O(N)$	$O(N^{\frac{3}{2}})$	$O(N^2)$
memory	$O(N)$	$O(N \log N)$	$O(N^{\frac{4}{3}})$

Existing Fast Solution Techniques

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  $\Rightarrow$ **Method of polarized traces**

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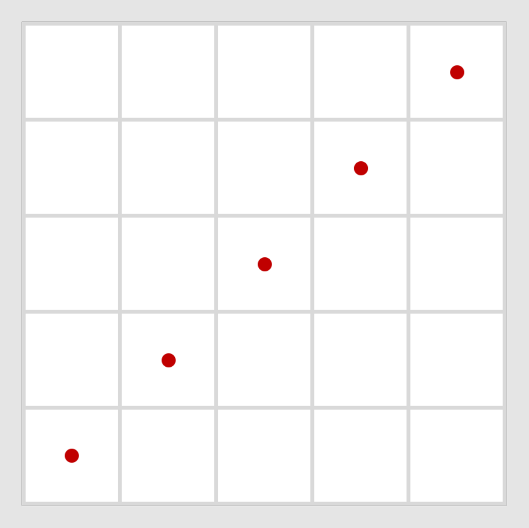
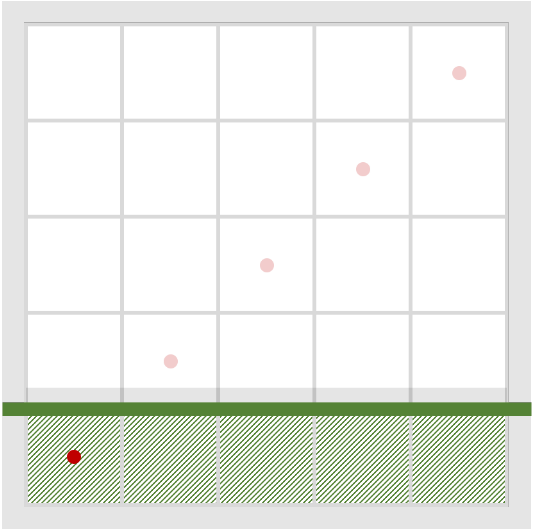
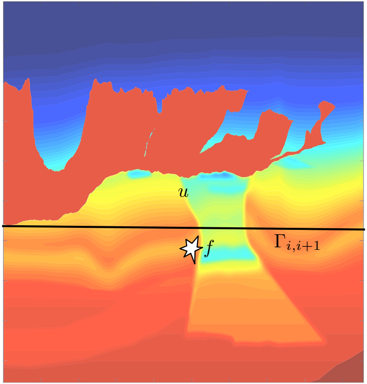
  $\Rightarrow$ **Method of polarized traces**



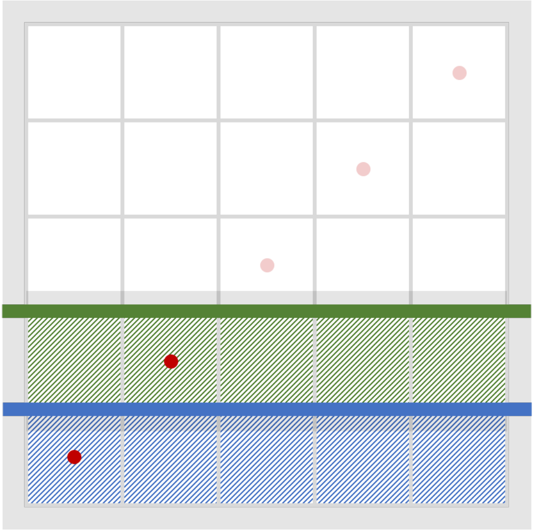
Method Of Polarized Traces

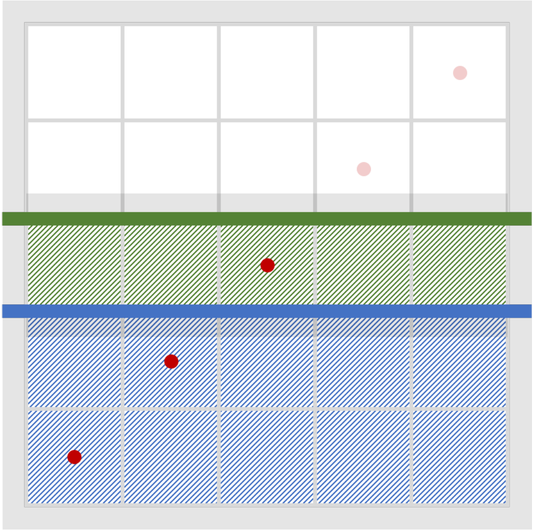


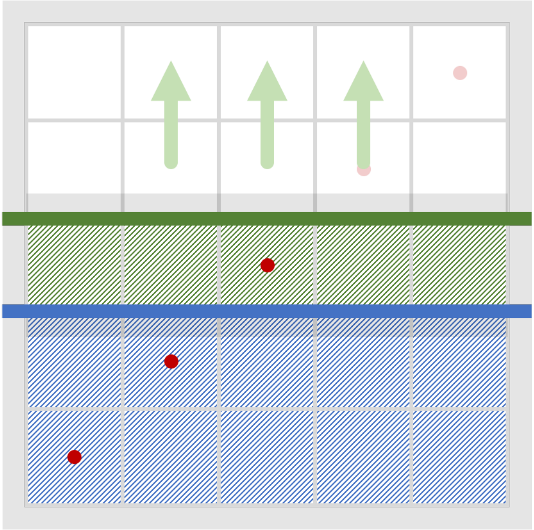


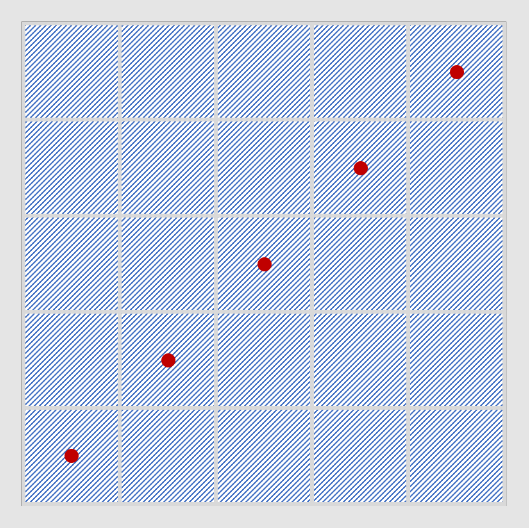
Polarization condition:

$$0 = -\int_{\Gamma} G(x,y)\partial_{n_y} u^{\uparrow}(y) ds_y + \int_{\Gamma} \partial_{n_y} G(x,y) u^{\uparrow}(y) ds_y$$

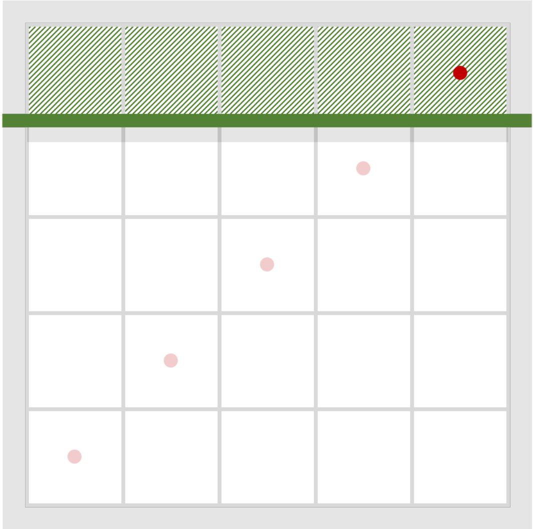








Method Of Polarized Traces



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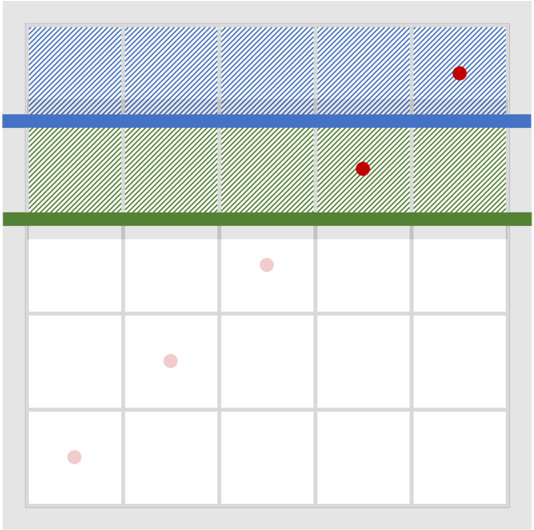
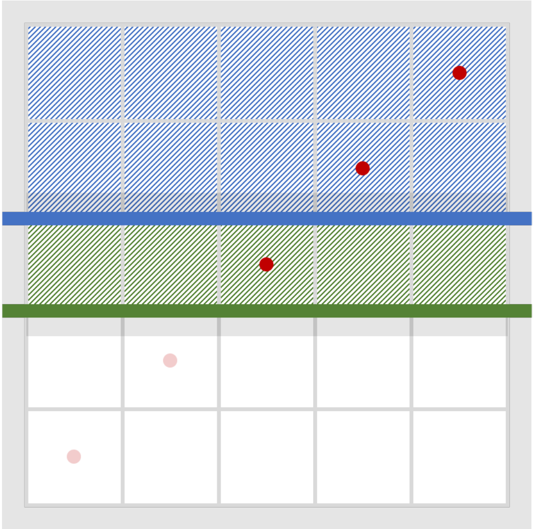
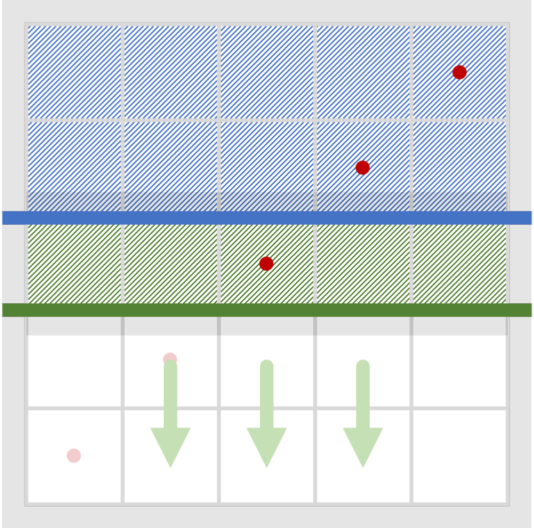
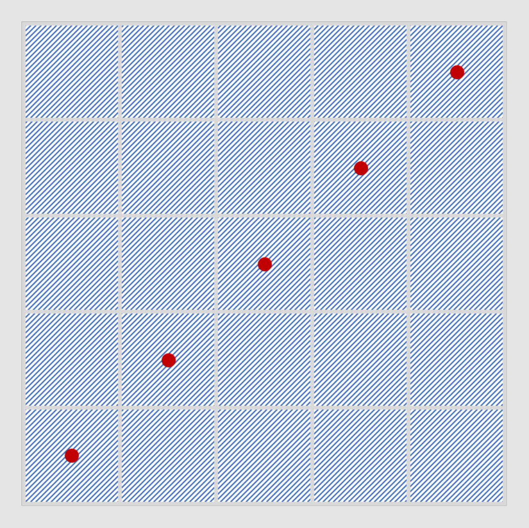




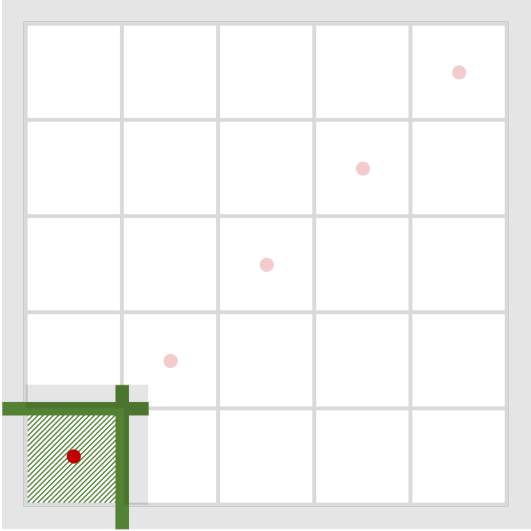




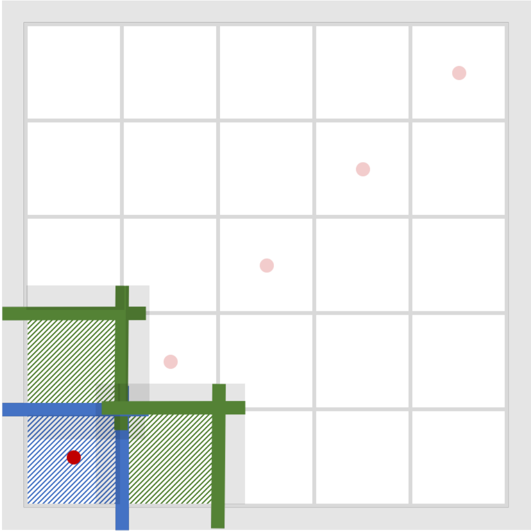
Serial complexity: $O(N)$

Question: Can we parallelize this preconditioner?

Problem: Serial nature of the sweeps



Solution: L-sweeps



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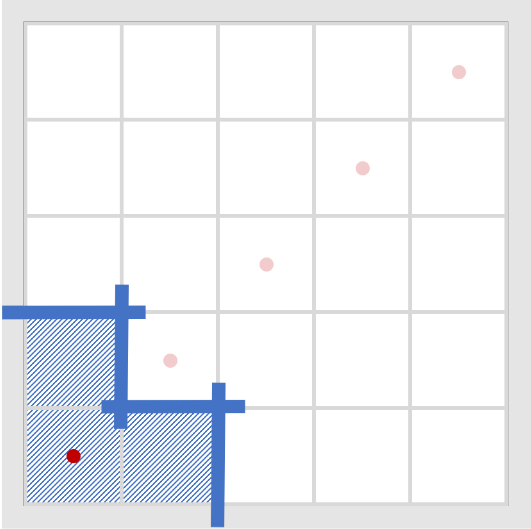

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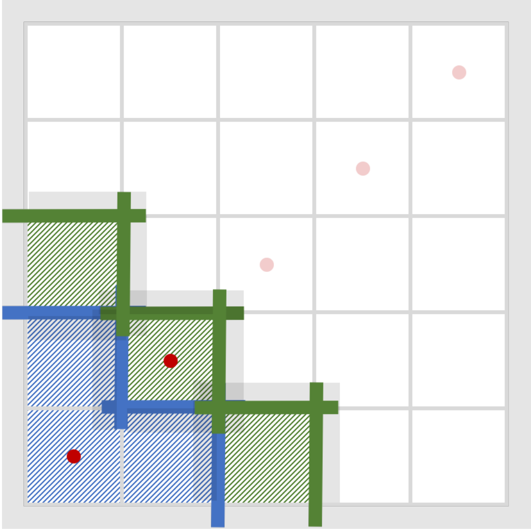
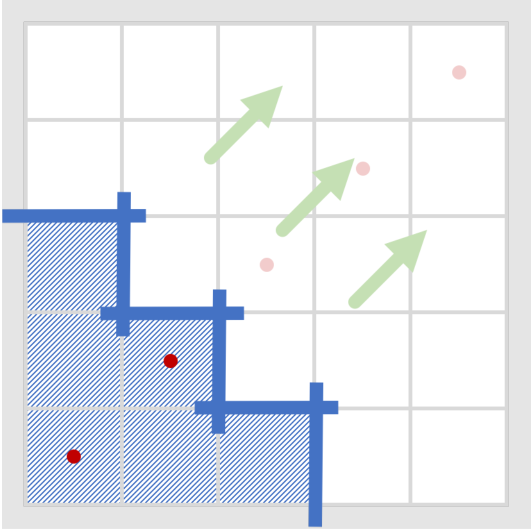


Solution: L-sweeps

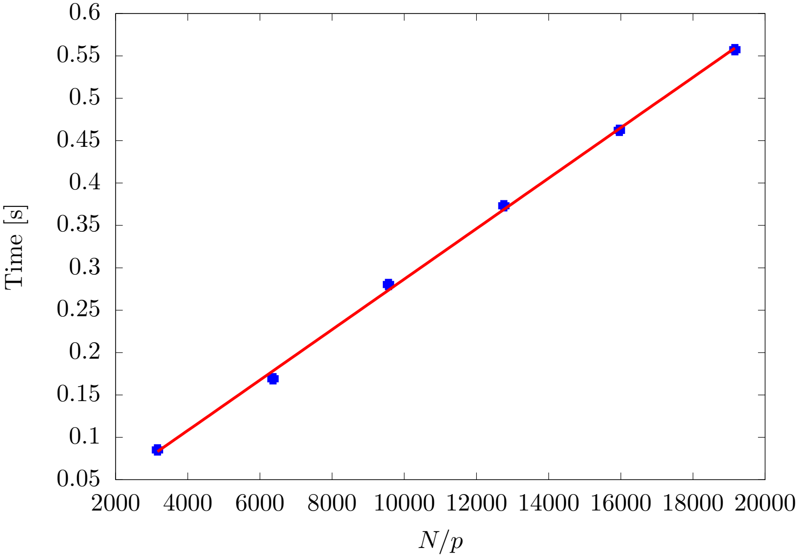


M O V I E! :)

Each propagation onto the next diagonal can is embarrassingly parallel on a cell-wise level!

$\Rightarrow O(N/p)$ complexity
(as long as $p = O(N^{1/d})$)

Numerical Example: Complexity



4 points per wavelength

Wavelengths in domain	Number of cells	Wavelengths in PML: 1	1.5	2	2.5	3
16	2	5	3	3	3	3
32	4	7	5	5	5	5
64	8	7	6	6	6	6
128	16	9	6	7	7	7
256	32	12	9	7	7	7
512	64	17	11	8	9	8
1024	128	29	14	11	9	9

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512	64	17	11	8	9	8
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Numerical Example: Iteration Count

6 points per wavelength

Wavelengths in domain	Number of cells	Wavelengths in PML: 1	1.5	2	2.5	3
16	2	4	3	3	3	3
32	4	5	3	3	3	3
64	8	7	3	3	3	3
128	16	9	5	4	3	3
256	32	11	6	5	5	4
512	64	17	9	7	5	5
1024	128	32	11	8	7	6

8 points per wavelength

Wavelengths in domain	Number of cells	Wavelengths in PML: 1	1.5	2	2.5	3
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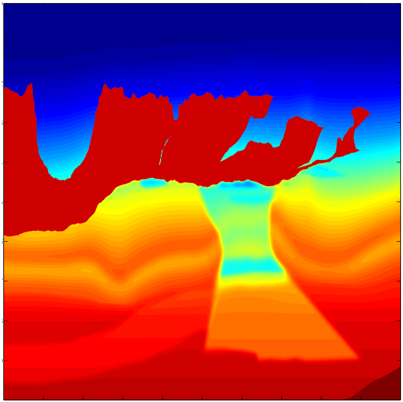
64	8	7	3	3	3	3
128	16	8	5	3	3	3
256	32	11	6	5	3	3
512	64	19	8	6	5	4
1024	128	-	11	9	7	5

Numerical Example: BP Model Setup



- Second order finite difference discretization
- unit square

4 points per wavelength

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16	2	9	7	6	6	6
32	4	12	7	7	7	7
64	8	14	9	10	10	10
128	16	16	12	12	12	12
256	32	25	25	23	22	23
512	64	30	26	26	26	26
1024	128	-	29	29	28	28

Numerical Example: Iteration Count

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16	2	9	6	6	6	6
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64	8	13	9	8	8	8
128	16	16	11	11	11	11
256	32	24	18	18	19	18
512	64	-	25	25	24	24
1024	128	-	29	28	28	27

Numerical Example: Iteration Count

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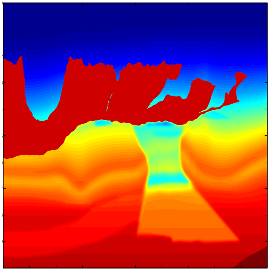
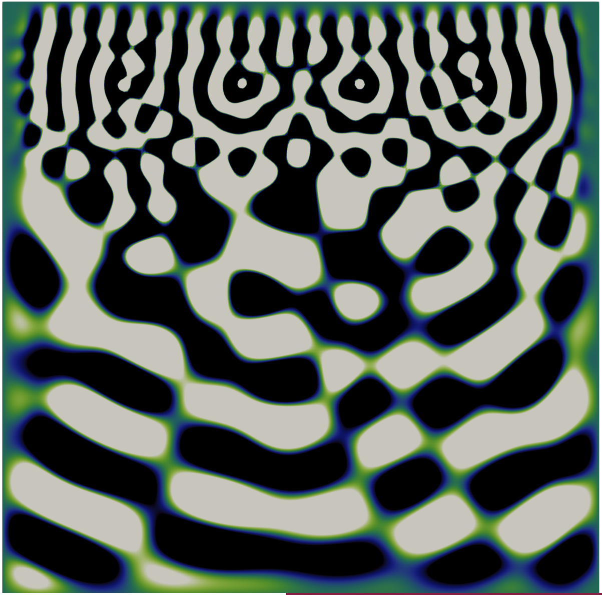
64	8	14	9	9	9	9
128	16	22	16	17	14	13
256	32	-	16	16	15	15
512	64	-	22	21	21	21
1024	128	-	-	26	26	26

Numerical Example: High Frequency Solutions



- max. 16 wavelengths in domain
- PML width: 1.25 wavelengths
- 2×2 domain decomposition



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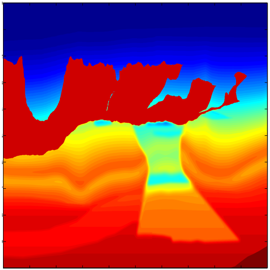
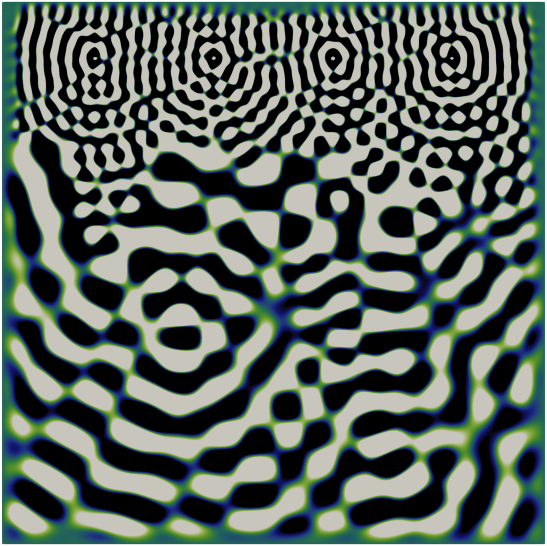


- max. 32 wavelengths in domain
- PML width: 1.5 wavelengths
- 4×4 domain decomposition



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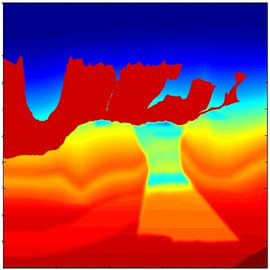
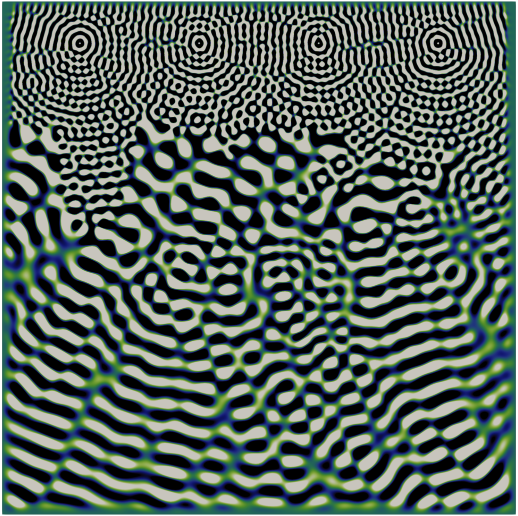
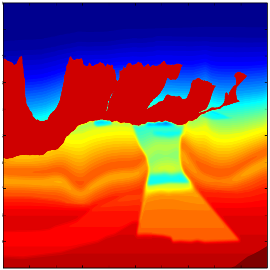


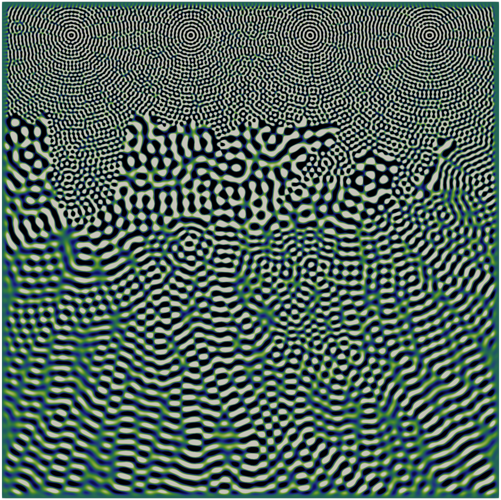
- max. 64 wavelengths in domain
- PML width: 1.75 wavelengths
- 8×8 domain decomposition



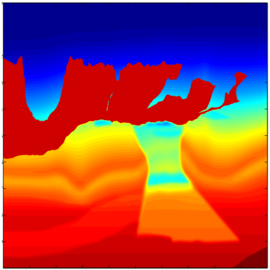
Numerical Example: High Frequency Solutions



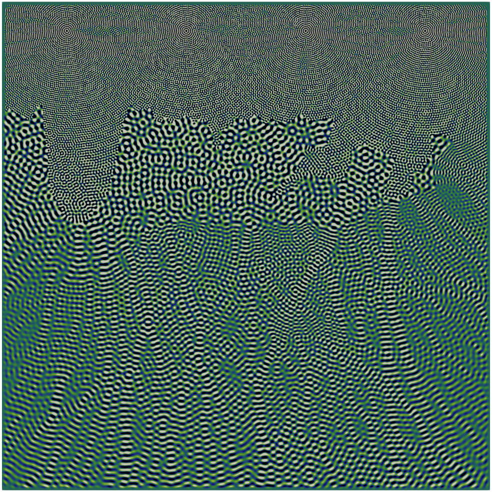
- max. 128 wavelengths in domain
- PML width: 2.00 wavelengths
- 16×16 domain decomposition



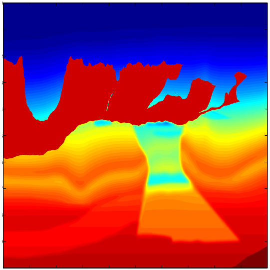
Numerical Example: High Frequency Solutions



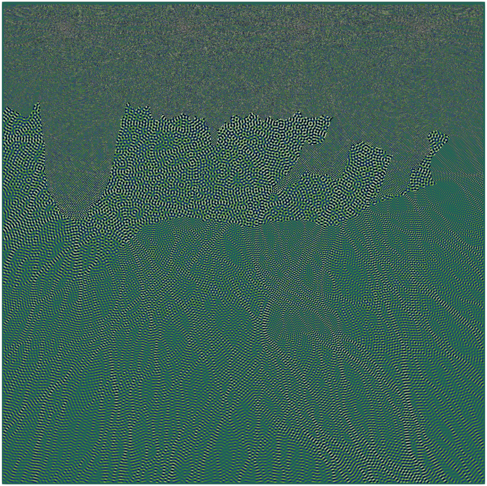
- max. 256 wavelengths in domain
- PML width: 2.25 wavelengths
- 32×32 domain decomposition



Numerical Example: High Frequency Solutions



- max. 512 wavelengths in domain
- PML width: 2.5 wavelengths
- 64×64 domain decomposition



Successful construction of a scalably parallelizable preconditioner for the high-frequency Helmholtz equation.

- $O(N/p)$ complexity as long as $p = O(N^{1/d})$
- Independent of the discretization
- Applicable to heterogeneous media

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Next steps:

- $O(N/p)$-scaling in 3D where $p = O(N^{2/3})$
- several right-hand sides ($O(1)$ scaling per right hand side?)